Taxation and Aggregate Factor Supply: Preliminary Estimates

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The life-cycle supply of the factors of production plays a crucial role in the analysis of many important economic problems. Among these are the optimal tax treatment of capital and labor incomes, the appropriate social rate of discount to be used for public projects, and the construction of indexes of economic welfare. Consider, for example, the simple model

\[ U = U(C_w, C_r, L_w) \]  

(1)

where \( U \) is a well-behaved utility function depending upon \( C_w \), which is the annual flow of consumption during working years, \( C_r \), is the annual flow of consumption during retirement, and \( L_w \), is the annual flow of leisure during working years.

It then follows from recent analytical results \(^1\) that the efficient (in the sense of minimizing the dead weight loss from raising a given tax revenue) taxation of \( C_w \) and \( C_r \) (\( L_w \) is assumed to be inherently too costly to attempt to tax) requires heavier taxation in the period in which consumption is a weaker substitute for leisure. The tax rates on \( C_w \) and \( C_r \) will be equal only if utility is

\(^1\) See, for example, Atkinson and Stiglitz, (1976).

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separable between goods and leisure; in general, the tax rates will be given by

\[ \frac{\hat{t}_w}{\hat{t}_r} = \frac{N_{21} + N_{12} + N_{23}}{N_{21} + N_{12} + N_{13}}, \]

where \( N_{ij} \) is the compensated cross elasticity of demand of \( i \) with respect to the price of \( j \) (\( 1 = C_w, 2 = C_r, 3 = L_w \)). Hence, the optimal tax rate on capital income will be positive (\( t_w < t_r \)) or negative (\( t_w > t_r \)) as \( N_{13} \) is larger or smaller than \( N_{23} \). Thus, the efficient taxation of capital income depends upon whether or not leisure and consumption during working years are complements, or at least weaker substitutes, than leisure and retirement consumption.

Consider next the appropriate social rate of discount. Recently, Harberger (1968) and Sandmo and Dreze (1971) have derived the result that the appropriate social rate of discount is a weighted average of the gross of tax marginal product of private capital and the net of tax rate of return to saving, the weights depending upon the interest elasticities of investment and saving, respectively. Since business and personal income taxes cause the two rates to diverge sharply, a positive interest elasticity of saving would result in a much lower social rate of discount than the gross private marginal product of capital. In the framework discussed above, the issue is the elasticity of \( C_w \) with respect to the forward price of \( C_r \).

Also of interest is the almost total lack of attention given to future consumption in the calculation of indexes of economic welfare. For example, saving is generally ignored in the calculation of constant utility index numbers, i.e., conceptually they often should be calculated as constant lifetime utility rather than as constant instantaneous utility.

Fortunately, the empirical analysis of factor supply has received renewed attention in recent years. The seminal work of Friedman (1957), Modigliani and Brumberg and Ando and Modigliani (1954, 1963) on life-cycle consumption and of Becker (1964, 1965) on labor supply (broadly construed to include human investment and nonmarket work) rekindled interest among em-

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\(^2\) See Harberger, (1964). If the two periods are of different lengths (e.g., a work life of 40 years and a retirement life of 20 years), the formula would be weighted to reflect this difference.

\(^3\) In the alternative approach which shadow prices public investment funds and discounts at the consumption rate of interest (e.g., Feldstein, 1974b), the shadow price depends on these elasticities.
pirical economists on these issues. Curiously, however, empirical studies of the role of interest rates or forward prices on consumption are few and far between. Most such studies conclude that interest rates have only a negligible effect on consumption or saving. Boskin (1976) has recently criticized much of this work, especially the structural interpretation of "Denison's Law," the alleged insensitivity of the saving rate to any economic variables (at full employment). He employed a variety of data sources and advances in econometric techniques to estimate an elasticity of private saving with respect to the real net rate of return of approximately 0.4.

The purpose of the present paper is threefold: first, to see if this sensitivity of saving with respect to the real net rate of return continues to obtain when explicit account is taken of leisure demand and also of the potential influence of social security; second, to estimate the parameters of the demand functions expressly derived from maximizing a utility function of the form of equation (1); and, third, to begin to account expressly for such life-cycle phenomena as changes in the age distribution of the population. We hope to be able at a later date to use the estimates to draw some provisional conclusions about the issues discussed earlier.

Toward this end, we present in the next section our basic model, its properties and estimating equations.

Following that is a discussion of the data used in this study, i.e., the national income and wealth accounts developed by Christensen and Jorgenson (1972) and estimates of forward prices, etc., developed by the authors.

A fourth section reports our preliminary empirical results, which are indeed striking. While many refinements are still to be made, our preliminary results strongly reject the structural interpretation of "Denison's Law." Our estimates suggest that the consumption saving choice is strongly influenced by relative prices, including the forward price of future consumption. Our estimates, which appear to be measured quite precisely, suggest that leisure and future consumption are much stronger substitutes than leisure and current consumption.

We conclude with some provisos and a long list of extensions, refinements, and improvements we hope to make relative to these preliminary results.

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4 Thus, Break (1974) notes "Unfortunately, empirical evidence on the interest elasticity of the saving rate is rare."
The General Model

The Direct Utility Function

We assume that each consumer unit (an individual or a household) has a utility function of the form

$$ U(L_1, C_1, L_2, C_2, B; t, A, D, T(A, t) - A, R(t) - A, s), $$

where $L_1 =$ quantity of leisure in period 1,
$L_2 =$ quantity of leisure in period 2,
$C_1 =$ quantity of consumption in period 1,
$C_2 =$ quantity of consumption in period 2,
$B =$ quantity of bequest,
t =$ calendar time,
$A =$ age,
$D =$ size of the household in equivalent adult consumption units,
$T(A, t) =$ life expectancy of an individual (or head of household) at age $A$ in year $t$,
$R(t) =$ expected retirement age at time $t$,
and $s =$ sex of individual or head of household.

As a first pass, we make three simplifying assumptions:
1. $R(t)$ is given exogenously, that is, it is not a choice variable of the consumer unit.
2. $L_2$ is assumed to be fixed and equal to $L$, that is, the consumer unit supplies no labor in the second (postretirement) period.
3. $B$ and $C_2$ will be treated as a composite commodity.

Utility Maximization

The consumer unit is assumed to maximize utility with respect to $L_1$, $C_1$, and $C_2$, subject to a wealth constraint:

$$ p_1 C_1 + p_2 C_2 = (1 - \mu) w_1 (\bar{L} - L_1) + W_p + (1 - \lambda) W_s, $$

where

$w_1 =$ wage rate,
$\mu =$ effective tax rate on labor income,
$p_1 =$ price of consumption in period 1,
$p_2 =$ forward price of consumption in period 2,
$W_p =$ private wealth,
$W_s =$ social security wealth,
and $\lambda =$ "discount" factor associated with social security wealth.
As usual, one can transform the budget constraint into the canonical form

$$w^*L_1 + p_1^*C_1 + p_2^*C_2 = 1,$$

where

$$w^* = \frac{(1-\mu) w_i}{(1-\mu) w_i L + W_p + (1-\lambda) W_s},$$

$$p_1^* = \frac{p_1^*}{(1-\mu) w_i L + W_p + (1-\lambda) W_s},$$

and

$$p_2^* = \frac{p_2^*}{(1-\mu) w_i L + W_p + (1-\lambda) W_s}.$$

The Indirect Utility Function

Under mild regularity conditions an indirect utility function of the form

$$V(w^*, p_1^*, p_2^*, t, A, D, T(A,t) - A, R(t) - A, s)$$

exists. If one specifies an indirect utility function, then the demand functions are given through Roy’s Identity as

$$L_1 = \frac{\partial V}{\partial w_1^*} = \frac{\partial V}{\partial w_1^* w_1^*} + \frac{\partial V}{\partial p_1^* p_1^*} + \frac{\partial V}{\partial p_2^* p_2^*},$$

and

$$C_i = \frac{\partial V}{\partial p_i^*} = \frac{\partial V}{\partial w_1^* w_1^*} + \frac{\partial V}{\partial p_1^* p_1^*} + \frac{\partial V}{\partial p_2^* p_2^*}, \quad i = 1, 2.$$

If we assume that the indirect utility function has the homogeneous translog form, we obtain the linear logarithmic expenditure system of Lau and Mitchell (1971), which gives the expenditure shares as

$$p_{ix}^* x_i = \alpha_i + \sum_{j=1}^{3} \beta_{ij} \ln p_j^* + \beta_{it} t + \beta_{iA} A + \beta_{iR} [T(A, t) - A] + \beta_{is} s, \quad i = 1, \ldots, 3, \quad (3)$$

where the subscripts 1, 2, and 3 stand for $L_1$, $C_1$, and $C_2$, and
\[ \sum_{i=1}^{3} \alpha_i = 1; \sum_{i=1}^{3} \beta_{ij} = 0, \forall j; \sum_{i=1}^{3} \beta_{it} = 0; \sum_{i=1}^{3} \beta_{iA} = 0; \]

\[ \sum_{i=1}^{3} \beta_{IT} = 0; \sum_{i=1}^{3} \beta_{iR} = 0; \sum_{i=1}^{3} \beta_{iu} = 0 \text{ and } \beta_{ij} = \beta_{ji}, \forall i, j. \]

The homogeneous translog indirect utility function is a homothetic utility function that implies that as total full wealth increases, holding prices constant, all expenditures will increase proportionally. Whether this is realistic is, of course, an empirical proposition. However, this assumption may be relaxed through the introduction of nonhomogeneity parameters such as \( \tilde{L} \) and \( \tilde{C}_i \) which may be estimated by iterative or search methods. On the first pass we maintain the homogeneity assumption; however, we will relax it in subsequent work.

To implement econometrically the model we have described, we need to be able to relate the current consumption of each age cohort, which is directly observable, to the annualized consumption flow during working life. One can then aggregate across age cohorts to obtain the aggregate current consumption which is again directly observable. Aggregate current saving then consists of two parts: planned consumption during the remainder of the working life and planned consumption (plus bequest) during the retirement period, both aggregated across age cohorts. Aggregate current consumption will then depend on the same variables which affect the consumption choices of each age cohort and in addition will depend on the age distribution of the population in the economy. As a first pass we make the simplifying assumption that the form of the aggregate current consumption function is the same as the individual age-cohort specific consumption functions (3). In a subsequent paper we shall attempt to derive the aggregate current consumption functions as the sum of individual age-cohort specific consumption functions rigorously.5

Data and Estimation Methods

The data used in this study come from a variety of sources reporting on aggregate U.S. time series from 1929 to 1969. Most of the data are derived from the complete-and-consistent-accounting system for the private sector of the U.S. economy developed by Christensen and Jorgenson (1972). These data include informa-

5 See Lau (1977) for a discussion of aggregation across consumers with different attributes.
tion on private income, consumer expenditure, labor compensation, property compensation, rates of return on capital, etc. We exclude expenditure on durables and include the rental flow on durables in consumption. We also have developed series on average household size, average retirement age, life expectancy, human investment, the age composition of the population, the sex composition of the labor force, etc. We are using these data to attempt to generate more reasonable life-cycle variables than are usually used in estimation of consumption functions.

The results reported below, as a first step, are intended only to compare our estimation methods with the usual analysis. We will report our estimates with the “life-cycle” adjusted variables in a subsequent paper. We shall also report results that deal explicitly with human capital accumulation. The results reported here, like all consumption or saving function estimates that are used to analyze aggregate capital accumulation, implicitly assume arbitrage between human and nonhuman capital formation.

The definitions of the key variables used in the analysis are as follows:

Current consumption: as mentioned previously, we exclude durables expenditure and include their rental flow; these and all other aggregate values are in real per capita terms.

Wealth: Ando-Modigliani market value of private nonhuman capital stock.

Social security wealth: Feldstein’s social security wealth series, as used by Barro (1977).

Wage rates: after-tax wage rates, adjusted for changes in labor force composition; see Christensen and Jorgenson (1972).

Forward price of future consumption: the forward price of future consumption is defined as

\[ P_2 = P_1 e^{-rT} \]

where \( r \) is the real after-tax rate of return on capital, \( P_1 \) is the current price index, and \( T \) is the length of time between saving and dissaving. Boskin (1976) discusses alternative estimates of the expected, longrun, real after tax rate of return on capital. Alternative estimates yield results similar to those reported below. In the results reported below, \( T \) is taken to be 20 years. The results are quite robust with respect to variations in \( T \) in the 10- to 30-year range.

Leisure consumption: annual hours available minus average hours of work. In the estimates reported below, available hours are set at 4,000; we also allowed this parameter to vary in increments of 500 hours, with results similar to those reported below.
Ideally, the hours available represent total hours in a year less time necessary to maintain human capital (e.g., sleeping and eating). Further, we are unable with these data to separate nonmarket work from true leisure, and this should be kept in mind in interpreting the results below.

Further, recall that these are preliminary estimates. Our goal, ultimately, is to develop and implement more theoretically sound measures of the life-cycle variables under study, including human investment, permanent income or wealth, etc. In the present paper our goal is the much more modest one of analyzing labor supply and consumption decisions jointly. Since many consumption and labor supply functions have been reported that ignore cross-price effects, we hope to begin to examine the full set of own- and cross-price effects. After all, one reason for working is to earn income in order to save for future consumption (especially for retirement).

The estimation of the system of consumer demand equations discussed earlier imposes several restrictions on the coefficients. In particular, adding up

$$\sum_j \beta_{ij} = 0$$

and symmetry

$$\beta_{ij} = \beta_{ji}$$

are to be imposed. The latter restriction constrains the same parameters in separate equations to the same values. We employ an iterative search procedure to estimate these parameters by least squares. Of course, the prices and income are potentially endogenous. Indeed, Boskin's (1976) estimates of the interest elasticity of saving double when he allows for such endogeneity. We shall report the results of two-stage least-squares estimates of our translog demand system in a subsequent paper.

With these provisos concerning the data and estimation methods in mind, we turn to a discussion of our empirical results.

**Empirical Results**

Our preliminary empirical results, estimates of the parameters of consumer demand equations based on aggregate U.S. time series data, are presented in table 1. The first part of table 1 reports our results excluding any wealth effect of social security. The equations perform quite well by conventional measures. The estimated standard errors of each regression are a small fraction of the mean values of the dependent variables. The parameter estimates do not
### Table 1.—Preliminary estimates of aggregate factor supply equations

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Social Security wealth excluded $(\lambda=1)$</th>
<th>Social Security wealth included $(\lambda=0)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_1$</td>
<td>0.2138 ($0.0196^*$)</td>
<td>0.1762 ($0.0138^*$)</td>
</tr>
<tr>
<td>$\beta_{LL}$</td>
<td>0.0875 ($0.0062^*$)</td>
<td>0.0812 ($0.0041^*$)</td>
</tr>
<tr>
<td>$\beta_{lL} = \beta_{1L}$</td>
<td>-0.1697 ($0.0126^*$)</td>
<td>-0.1332 ($0.0106^*$)</td>
</tr>
<tr>
<td>$\alpha_2$</td>
<td>0.6138 ($0.0467^*$)</td>
<td>0.5271 ($0.0529^*$)</td>
</tr>
<tr>
<td>$\beta_{12}$</td>
<td>0.0270 ($0.0338^*$)</td>
<td>0.0353 ($0.0399^*$)</td>
</tr>
<tr>
<td>log of likelihood function</td>
<td>105.08</td>
<td>108.85</td>
</tr>
</tbody>
</table>

Note: $\beta_{l2} = \beta_{21} = -(\beta_{LL} + \beta_{21}); \beta_{12} = \beta_{21} = -(\beta_{LL} + \beta_{12})$.

Note: Test of no effect of social security:

$$-2 \ln \lambda = 7.54 > \chi^2_{0.05} = 3.84.$$ 

* Estimated standard error in parentheses.

Violate any restrictions of consumer theory. The estimated standard errors for almost all of the coefficients are less than one-tenth of their respective point estimates, i.e., virtually all of the own- and cross-price effects and expenditure shares are estimated quite precisely.

The second part of table 1 reports our results including full social security wealth in the budget constraint, i.e., $\lambda = 0$. Again, the estimated standard errors of the regression are a minute fraction of the mean values of the dependent variables. Almost all of the coefficients are estimated quite precisely. Comparing the coefficients, we see that the introduction of social security wealth does not change the estimated own- and cross-price effects very much.

It is instructive to test formally the hypothesis of no social security effect on consumption or labor supply during the period under study. This may be done via the usual likelihood ratio test. The test of $\lambda = 1$, no effect of social security, vs. $\lambda = 0$, is rejected with a $\chi^2$ statistic twice the critical value. We also allowed $\lambda$ to vary from 0 to 1 in increments of 0.1. The maximum value of the likelihood function occurred at $\lambda = 0$. While these data suggest that social security did indeed influence aggregate factor supply in the
period between the 1930's and 1960's, two cautionary notes are in order. First, our analysis suggests that social security affected aggregate factor supply. Feldstein (1974a) suggests that social security reduced private saving. Boskin (1977) suggests it accelerated earlier retirement. Our results do not distinguish between effects on labor supply and consumption, and hence cannot be viewed as support for either of these propositions.

Second, and conceptually much more important, these data refer to the period when social security was first starting, during which time an enormous intergenerational transfer of resources was made to the first few cohorts of retirees under social security. It may well be that there was a corresponding reduction in private transfers, as discussed, for example, by Barro (1975). A large percentage of such transfers might well have occurred outside normal market processes and so may have escaped the usual types of national income accounting. More importantly, once the social security system matures, it may well be that each cohort will get back in benefits approximately its contribution plus interest, and that social security will have no additional effect whatsoever on private savings decisions. Extrapolating from the transition phase may therefore be quite misleading.

Since it is not easy to interpret the coefficients of the translog consumer demand system, we present in table 2 some of the more interesting own- and cross-price elasticities. These are functions of the parameters, and we may thus compute their point estimates and estimated asymptotic standard errors. Their estimated t ratios range from 5 to 40, so all are measured quite precisely.

Note first that the forward price of future consumption, which is developed from estimates of the longrun expected real net of tax rate of return on capital, has a substantial effect on current consumption. When translated into the more usually discussed net real rate-of-return elasticity of private saving, our estimate is 0.4. This is quite similar to Boskin's (1976) estimates of consumption functions only. While hardly enormous, this estimate substantially exceeds the commonly accepted conjecture in the "Denison's Law" tradition of a zero elasticity of the private saving rate with respect to any economic variable at full employment.

Even this modest interest elasticity of saving has drastic implications for such issues as the relative efficiency and incidence of income and consumption taxes, the appropriate social rate of discount, etc. These are discussed in some detail by Boskin (1976).

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6 Evaluated at 1955 values of the variable. The elasticities are quite similar if evaluated at, say, 1969 values.
Table 2.—Estimated elasticities

<table>
<thead>
<tr>
<th>With respect to:</th>
<th>$C_1$</th>
<th>$C_2$</th>
<th>Labor</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W$</td>
<td>0.28</td>
<td>1.11</td>
<td>-0.08</td>
</tr>
<tr>
<td></td>
<td>(3.07)</td>
<td>(39.46)</td>
<td>(-15.64)</td>
</tr>
<tr>
<td>$P_1$</td>
<td>-0.89</td>
<td>0.28</td>
<td>0.18</td>
</tr>
<tr>
<td></td>
<td>(-6.75)</td>
<td>(3.79)</td>
<td>(10.88)</td>
</tr>
<tr>
<td>$P_2$</td>
<td>0.52</td>
<td>-1.49</td>
<td>-0.08</td>
</tr>
<tr>
<td></td>
<td>(2.87)</td>
<td>(-15.71)</td>
<td>(-3.93)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>-0.54</td>
<td>1.54</td>
<td>0.08</td>
</tr>
<tr>
<td></td>
<td>(-2.87)</td>
<td>(15.71)</td>
<td>(3.93)</td>
</tr>
</tbody>
</table>

1 $t$-statistics in parentheses.
2 Allowing income to vary with $R$. The elasticities with respect to $R$ holding full permanent income constant are similar, but slightly larger in absolute value.

Note also the fact that future consumption is much more price elastic than present consumption and that the cross elasticities, while much smaller than the own elasticities, are hardly negligible. While much more work remains to be done, as discussed later, these results not only hint that heavy taxation of capital is inefficient, but also suggest that the optimal tax rates (see Stiglitz and Boskin, 1977) may even call for heavier taxes on current than on future consumption (i.e., a consumption tax combined with an interest income subsidy).

We note further that the coefficients imply a slightly backward-bending labor supply function. The total wage elasticity is about -0.08. This estimate is somewhat less backward-bending than the estimates of many others (see, e.g., Ashenfelter and Heckman, 1973, and Harberger, 1964). Of course, the pure substitution elasticity is modestly positive even in these aggregate data. This may be due to the corrections for labor force composition; in view of the substantial evidence of large labor supply elasticities estimated from household data for wives and the elderly (see, e.g., Boskin, 1977; Rosen, 1976; Heckman, 1974; etc.), we shall not push this point too far. Also note the nontrivial cross-interest rate elasticity for labor supply. While it is unusual to include interest rates (or forward prices of future consumption) in labor supply functions, it is not unreasonable to believe that many workers are earning a substantial share of their marginal earnings in order to save for future consumption. These results suggest that previous estimates of both labor supply functions and consumption functions have been misspecified since they exclude the cross-price terms.
Conclusions and Caveats

Our preliminary results prove quite interesting and promising. We cannot emphasize too strongly that these results are preliminary. Our ultimate goal is to develop what might best be called life-cycle national accounts (as opposed to current accounts), to incorporate human capital explicitly into the analysis, and to improve our analytic and econometric procedures. We feel that we have made some progress in the results reported here and that these are not without interest in and of themselves. We are also in the process of dealing with several other issues. One is better measurement of permanent income, consumption and saving in the face of changes in the (age and household) structure of the population and in the (sex) composition of the labor force which render current measures of these variables suspect. We are also studying the econometric problems of consistent aggregation, treatment of serial correlation in simultaneous equation systems, endogeneity of the prices (for a first step in this direction, see Boskin, 1976); explicit human investment functions; general non-homogeneity; more general models of expectations concerning future incomes; and, hopefully, the endogeneity of retirement, and the separating of bequests from future consumption.

References


Boskin and Lau have given us much to digest. Their point that demand elasticities are best estimated for complete systems hardly seems arguable. Further, their attempt actually to estimate this system (rather than merely suggest that it would be nice for someone to do so) deserves credit.

There is a real asymmetry in economics between doing and recommending. Recommending is virtually costless and, if a modicum of reason is used, riskless. Doing is costly, and it subjects the doer(s) to criticism of not having done it “right.” Oddly enough, economists in offering their criticisms tend to be very uneconomic: comparisons are made usually with the most complicated procedures using ideal data rather than with practical alternatives. So, in offering my own criticism of the Boskin-Lau work, which is based on few practical considerations, I at least honor a long tradition.

Chances are that I am overly critical, possibly for two reasons. First, as a novice to public-finance research, I have a better idea of how “it” should be done, if doing were costless, than of practical alternatives. Second, something can be said for criticizing while works are in process (as this one is) and reserving accolades for the finished product.

We are first warned in the title that estimates are preliminary. Then, results are dubbed preliminary (roughly) six times in the text, which, in addition, refers four or five times to modifications on the horizon. Just how preliminary these estimates are is noted by making the point that simultaneity has not been taken into account, yet in Boskin’s earlier work corrections for simultaneity doubled estimated interest elasticities of savings—the parameter of major interest in this study. From this, I assume that the paper is intended to be of interest more for its methods than for its estimates.

There are really two quite distinct parts to the paper. One consists of estimating a system of demand equations, while the other contains a discussion of how parameter estimates might be used for calculating optimal tax schemes.

It must come as a surprise to anyone to see (lifetime) utility specified as a function, as in equation (1), of rates of consumption.
during working and retirement years without reference to the length of the working and retirement period. Admittedly, the empirical model does acknowledge the length of these periods (current age, life expectancy, and expected age at retirement), but the theoretical model does not.

I, for one, am not convinced that the "optimal" taxation guidelines, equation (2), that emerge from second-order approximations to one-period static utility models give reasonable rules for taxation between periods. They would, of course, if period length were truly fixed—but it is not. Even if life expectancy and age of retirement were exogenous to the individual, there are variations among individuals. This is particularly true for the point-in-time aggregate perspective Boskin and Lau take. Their time-series analysis has changing age distributions, changing age of retirement distributions, and life expectancies behind it. Further there is endogeneity. If health economists are to be believed, life expectancy is itself endogenous, and if a well known authority (Michael Boskin) is to be believed, age at retirement is as well.

Actually, I do not quarrel with the empirical presumption that period lengths are exogenous—it seems reasonably expedient. I do, however, wonder how endogeneity and life-cycle issues affect the theory of optimal taxation. I don't think this is an easy problem, but I do think it is one worth solving before we seriously approach questions of differential taxes on post- and preretirement income.

The Empirical Model, Or, What Did They Do?

1. I know there is a tradition for it, but a time-series analysis of aggregate fluctuations in income and hours worked exclusively from a perspective of commodity demand doesn't sit well. I would guess that secular drift probably does measure consumer responses to expanding budgets. Even for the long run, wages and labor supply and savings and interest rates may be jointly determined. In the short run, e.g., over business cycles, hours worked may very well be more constrained than the long-run commodity demand perspective suggests.

2. Even conceding exogeneity of the length of the work and retirement periods, the fact that these variables were introduced as controls in the regressions suggests that periods vary. Is it plausible to surmise (as does the empirical model) that substitution parameters are independent of period lengths? Specifically, would
you suppose that the degree of substitutability between pre- and postretirement leisure varies depending on how "close" one is to retirement? If so, then aggregate substitution parameters are sensitive to age distributions.

3. Often in empirical work an unknown, essentially a nuisance parameter, appears nonlinearly to confound estimation. In this study there are three such unknowns: (a) the maximum potential quantity of leisure, $\bar{L}$; (b) the social security wealth discount factor, $\lambda$; and (c) the interval between the work and retirement period, $T$, which is used to compute the forward price of retirement consumption. The typical empirical procedure is to pick reasonable values for these parameters and to estimate conditionally on them. The second part of this procedure is to assert that other parameter estimates are insensitive to the particular values of nuisance parameters. But how low is low? It would be helpful if alternative estimates (together with logarithmic values of likelihood functions) were shown. Further, the comment that with 0.1 increment between 0 and 1 on the discount parameter, $\lambda$, the likelihood function is maximized at 0 raises a question of the behavior of the likelihood function for negative values of $\lambda$.

(Maybe Social Security is the best of all sources of wealth).

4. The reference to simultaneity is unclear as I see it. Sources of simultaneity include:
(a) joint determination of aggregate labor supply and wage rates, and
(b) endogeneity in normalized prices through
- measurement errors in computing the denominator with elements $\mu$, $W_s$, $L$, $W_p$, $\lambda$, and $W_t$,
- true endogeneity in $W_p$ and (less plausibly) $W_s$, and, with progressive taxes together with options for income deferral,
- endogeneity of the price of future consumption.

If estimates are as sensitive to simultaneity corrections as the reference to Boskin suggests, then the job before Boskin and Lau is not trivial.

5. There are at least two ways to view constraints imposed by "the" theory. One, the one Boskin and Lau opt for, is to use the constraints to restrict parameter space and, therefore, to enhance estimation efficiency. The other is to use the data to test the theory:

1 Actually, since current age, age at retirement, and life expectancy are introduced as controls, there is a question of why "$T$" had to be estimated.
If the theory suggests constraints, do the data agree? Since Boskin and Lau make the point that estimation errors are small relative to parameter estimates, and since the estimates are conditional on the symmetry ($\beta_{ij} = \beta_{ji}$) and demand-depends-on-relative-price ($\sum_j \beta_{ij} = 0$) constraints, it would be nice if these constraints were tested.

6. Finally, and most embarrassing for me, I really do not know how the system of equations referring to expenditure shares is estimated. The empirical theory implies three choice commodities—preretirement consumption, postretirement consumption, and preretirement leisure. Yet the data seem to refer to total consumption without pre- and postretirement distinctions, and tables 1 and 2 appear as though they are estimated for two commodities only (current consumption and preretirement leisure). While the third option can be eliminated via an exogenous budget, it would be useful to know how preretirement consumption and leisure are imputed.

Finally, my comments are largely matters of detail. Boskin and Lau tackle important issues, and their boldness is to be applauded. I hope that as the work proceeds, more details both of theory and of empirical construction will be supplied.