Housing Tenure, Uncertainty, and Taxation

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ABSTRACT

Modern empirical work on the choice between renting and owning focuses on the concept of the "user cost" of housing, which integrates into a single measure the various components of housing costs. The standard approach implicitly assumes that households know the user cost of housing with certainty. However, the ex post user cost measure exhibits substantial variability over time, and it is highly unlikely that individuals believe themselves able to forecast these fluctuations with certainty. In this paper, we construct and estimate a model of the tenure choice that explicitly allows for the effects of uncertainty. The results suggest that previous work which ignored uncertainty may have overstated the effects of the income tax system upon the tenure choice.

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I. Introduction

The personal income tax provisions associated with homeownership have come under increasing scrutiny by both policymakers and academic researchers. This renewed interest has come about primarily because of the tremendous acceleration of real house prices in the past decade and the belief that Americans invest "too much" in owner-occupied housing relative to investment in more productive plant and equipment. (Feldstein [1982], Hendershott [1982].) Both phenomena are blamed in part on the interaction of inflation and the treatment of owner-occupied housing in the federal income tax system. The non-taxation of implicit rental income, the deductibility of nominal interest payments and property taxes, and the virtual exclusion of housing capital gains from taxable income are all believed to provide incentives for households to become owner-occupiers.

Modern empirical work on the choice between renting and owning focuses on the concept of the "user cost" of housing, which integrates into a single measure the various components of housing costs: interest rates, property and income taxes, maintenance, depreciation, expected capital gains, etc. A typical approach is to compute the \textit{ex post} value of the user cost of owner occupation each period, and then estimate a regression of the proportion of owner-occupiers in the population on the user cost and other variables. This approach has been fairly successful in explaining the movement of the homeownership ratio over time. (Rosen and Rosen [1980], Hendershott and Shilling [1980].)

The standard approach implicitly assumes that households know the user cost of housing with certainty. However, the \textit{ex post} user cost measure exhibits substantial variability over time, and it is highly unlikely that individuals believe themselves able to forecast these fluctuations with
certainty. Since housing decisions are usually made over time horizons of several years, this uncertainty can have important consequences for behavior. Ignoring it can lead to incorrect predictions of how people will behave under certain conditions. Consider these two examples:

1) During a period of time, housing prices increase substantially year after year. *Ex post* measures of the user cost of owner-occupation suggest that families should become homeowners in order to reap the capital gains. However, individuals do not know *ex ante* that these gains will occur. Indeed, past price increases may increase their subjective uncertainty concerning future movements in price. To the extent that they are risk averse, this increase in uncertainty will discourage people from becoming homeowners.\(^1\)

2) The government announces that it will begin taxing housing capital gains at the same rates as ordinary income. Focusing only on the *ex post* user cost suggests that such a policy will decrease the incidence of owner-occupation in the population. But the policy also lowers the variance of the user cost of homeownership—the government in effect becomes a silent partner, sharing both gains and losses. If individuals are risk-averse, this will tend to increase the attractiveness of owner-occupation, *ceteris paribus*.

In this paper, we construct and estimate a simple model of the tenure choice that explicitly allows for the effects of uncertainty. Section II presents the basic model and Section III discusses econometric issues involved in its estimation. Section IV presents the results and some of their implications. Price uncertainty is shown to have a statistically significant

\(^1\)In fact, during the 1970's, substantial increases in house prices occurred with barely any movements in the proportion of homeowners -- the figure in 1973 was 0.645; and in 1977, 0.648. (See Rosen [1981].) In the early 1980's, the proportion of homeowners declined from 0.658 in 1980 to 0.643 in 1982. (See U.S. Bureau of the Census [1980, 1982].)
and quantitatively large impact on the percentage of owner-occupiers. The results suggest that previous work which ignored uncertainty may have overstated tax effects on tenure choice. Section V provides a summary and suggestions for additional research.

II. The Model

In this section we develop a model of household tenure choice which focuses on the role of price uncertainty. Assume that an individual's utility depends upon his consumption of housing services and of a composite of all other goods. Housing services are assumed available in either of two mutually exclusive modes; renting or owning. For simplicity, renting and owning are modelled as distinct commodities with characteristics which may differ. For example, it may be difficult to rent a single unit with a large backyard. Similarly, it may be impractical for a homeowner to contract for the kind of maintenance services available to a renter.\(^2\) Algebraically, if \(G\) = quantity of the composite good, \(H\) = quantity of housing services consumed in owner-occupation mode, and \(R\) = housing services consumed in rental mode, then

\[
U = U(G, H, R)
\]

where \(U(\cdot)\) is the utility function, and \(H \times R = 0\).

At the time the tenure choice is made, the future real prices of both modes are uncertain. As will be shown below, the real cost of owner-occupation \((P)\) depends inter alia upon future housing capital gains, interest rates, and

\(^2\)Henderson and Ioannides [1983] provide a useful discussion of the distinctions between renting and owning.
federal income tax rates; none of which is known with certainty. Similarly, in the absence of long run indexed leases for rental housing, uncertainty also surrounds its real price ($\bar{Q}$). The price of the composite good is assumed to be known with certainty, and is equal to unity.

The individual makes his choice by comparing the outcomes of two subproblems. The first is maximizing utility, assuming that owner occupation is selected, and the second is maximizing utility assuming that renting is selected. Let $\tilde{V}^h(\bar{P}, y)$ be the maximum utility associated with owning, and $\tilde{V}^r(\bar{Q}, y)$ be the maximum utility associated with renting; where $y$ is permanent income over the planning period. An individual elects to own if:

$$E[\tilde{V}^h(\bar{P}, y) - \tilde{V}^r(\bar{Q}, y)] > 0.$$ 

Defining the expected prices of homeownership and renting as $\bar{P}$ and $\bar{Q}$, respectively, and taking a second-order Taylor series expansion of $\tilde{V}^h(\bar{P}, y)$ around the point $(\bar{P}, y)$ yields:

$$E[\tilde{V}^h(\bar{P}, y)] = \tilde{V}^h(\bar{P}, y) + \frac{1}{2} \tilde{V}^h_{11}(\bar{P}, y) \cdot \sigma^2$$

where $\tilde{V}^h_{11} = \frac{\partial^2 \tilde{V}^h}{\partial P^2}$ and $\sigma^2 = E(\bar{P} - \bar{P})^2$. Similarly:

$$E[\tilde{V}^r(\bar{Q}, y)] = \tilde{V}^r(\bar{Q}, y) + \frac{1}{2} \tilde{V}^r_{11}(\bar{Q}, y) \cdot \delta^2$$

where $\tilde{V}^r_{11} = \frac{\partial^2 \tilde{V}^r}{\partial Q^2}$ and $\delta^2 = E(\bar{Q} - \bar{Q})^2$. Hence, we can write:

$$E[\tilde{V}^h(\bar{P}, y) - \tilde{V}^r(\bar{Q}, y)] = \tilde{V}^h(\bar{P}, y) - \tilde{V}^r(\bar{Q}, y) + \frac{1}{2} \tilde{V}^h_{11}(\bar{P}, y) \cdot \sigma^2 - \frac{1}{2} \tilde{V}^r_{11}(\bar{Q}, y) \cdot \delta^2.$$

One thus expects that (to a second order approximation) the tenure choice will depend upon: i) the expected prices of the modes $(\bar{P}, \bar{Q})$, and ii) the variation of actual prices about the
These latter terms (referred to herein as the forecast error variances) figure importantly in our test of the relevance of uncertainty to tenure choice.

Our focus has been on the tenure choice at an individual level. Aggregation presents the usual difficulties, but may be motivated by considering a population with heterogeneous tastes and incomes, but identical expectations for future prices. For individual $i$, define

$$D^i(\Delta^i, y^i) = E(V^h,i - V^r,i)$$

where $\Delta^i$ is a vector of taste parameters. Integrating over the joint distribution of $y^i$ and $\Delta^i$ in the population for year $t$ yields the relation

$$\theta_t = \theta(\bar{P}_t, \bar{Q}_t, \sigma^2_t, \delta^2_t, y_t), \quad (2.1)$$

where $\theta_t$ is the aggregate proportion of homeowners.

For purposes of empirical implementation, a specific functional form must be adopted for (2.1). We assume the convenient specification

$$\ln \left[ \frac{\theta_t}{1-\theta_t} \right] = \beta_0 + \beta_1 \bar{P}_t + \beta_2 \bar{Q}_t + \beta_3 \sigma^2_t + \beta_4 \delta^2_t + \beta_5 y_t + \epsilon_t, \quad (2.2)$$

where $\epsilon_t$ is a random error. To the extent that changes in the prices of the respective tenure modes induce the same changes in utility levels, ceteris paribus, expected prices and forecast error variances affect the

Note that as a consequence of the assumption that renting and ownership are mutually exclusive, the covariance between the prices does not enter. Further, it is assumed that permanent income is independent of the prices $P, Q$.

It should be stressed that the variance terms are consequences of underlying uncertainty in the price of housing, and not the result of asset portfolio considerations. The interaction of housing and financial decisions is beyond the scope of this paper.
tenure decision in a symmetric fashion. Symmetry is clearly an empirical issue. For now, we merely note that it implies:

\[ \beta_2 = -\beta_1 \quad \beta_4 = -\beta_3 \]  
(2.3)

These restrictions will be tested below.

III. Empirical Implementation

We estimate equation (2.2) with annual U.S. data for 1956 to 1979. In this section we explain the construction of empirical counterparts to the theoretical constructs of Section II. The sources of all data are documented in an appendix available upon request.

1. The proportion of homeowners (\( \theta_p \))

Jaffee and Rosen [1979] argue that demographic changes in the U.S. population have had a major effect on the rate of household formation and homeownership, and that meaningful comparison of homeownership rates over time requires that such changes be taken into account. We adapt the Jaffee-Rosen procedure, which consists of creating a series which controls for the changing mix of household types due to changes in the age distribution of the population and alterations in marriage and divorce patterns.

2. The expected price of owner-occupation (\( \bar{P}_p \))

Computation of the price of owner-occupation is complicated by the fact that owners do not pay an explicit annual rent for housing. An important part of the annual cost of owner-occupied housing services is the unobservable

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5In terms of our theoretical model, a sufficient condition for symmetry is that \( V_h(\cdot) \) and \( V_F(\cdot) \) be identical up to an additive constant.
opportunity cost of the owner's equity in the house. Moreover, the federal income tax lowers the effective cost by allowing deductions of mortgage interest payments and local property taxes. Finally, like any other asset, anticipated capital gains on a house (either positive or negative) have an impact upon its effective rental price. Readers familiar with the neoclassical investment literature will recognize the similarity between constructing the price of owner-occupied housing services and the "user cost of capital." (See, e.g., Jorgenson [1971].)

The construction of user costs for housing is now familiar, and there is no need to go through the derivation again in detail. Let \( V_t \) = the market value of a house in period \( t \), \( r_{ct} \) = the individual's opportunity cost of capital, \( r_{mt} \) = the mortgage rate, \( D_t \) = depreciation, \( M_t \) = maintenance, and \( T_t \) = property taxes. If the share of owner's equity in the house is \( \gamma_t \), then the real annual cost of owner-occupied housing services in year \( t \), \( P_t \), is

\[
P_t = \frac{(1-\tau_t)(\gamma_t r_{ct} V_t + (1-\gamma_t) r_{mt} V_t + T_t) + D_t + M_t - V_t}{P_{L_t}}
\]

where \( \tau_t \) is the marginal income tax rate in period \( t \), \( V_t \) is the expected capital gain in period \( t \), and \( P_{L_t} \) is an index of the general price level.

Data on mortgage rates are not available for the entire sample period, nor is there sufficient information to allow calculation of \( \gamma_t \). We therefore

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7Dougherty and Van Order [1982] provide a careful derivation.

8Expression (3.1) ignores transactions costs. The fact that \( V_t \) is not multiplied by \( (1-\tau_t) \) reflects the reasonable approximation that housing capital gains are untaxed. (See Congressional Budget Office [1981].) \( V_t \) is measured as \((\hat{V}_{t+1} - V_t)\) where \( \hat{V}_{t+1} \) is the optimal prediction of \( V_{t+1} \) generated by an ARIMA model. In some versions, a separate ARIMA model is used to compute \( \hat{V}_{t+1} \); in others, the same ARIMA model is applied to all components of equation (3.1). As shown in Section IV below, the substantive results are essentially unchanged by the difference. (All expectations are computed in real terms.)
assume that $r_{mt} = r_{ct}$, which makes $\gamma_t$ irrelevant. For $r_{ct}$, the AAA corporate bond rate is used. No time series data are available on the depreciation and maintenance costs of the stock of owner-occupied housing. Following general real estate practice, we take depreciation and maintenance each to be 1 percent of the house value, $V_t$. Property taxes are computed as the average noncommercial property tax per owner-occupied dwelling. The term $\tau_t$ is the average marginal tax rate on income as calculated by Joines [1981]. $\pi_t$ is the implicit price deflator for total consumption expenditures with base year 1972. ($\pi_{1972} = 1.0.$)

Substituting all of these variables into (3.1) gives us only the ex post cost of owner-occupation in year $t$, while our theory suggests that tenure decisions are based upon the expected annual cost over the relevant horizon. Only if expectations are myopic will people expect the current real price to continue into the future. Because expected housing prices are not directly observable, they must be constructed on the basis of some model. There has been a long and sometimes acrimonious debate on just how expectations are formed. (Much of the discussion is reviewed by Friedman [1979].) We use

9 For a variety of reasons, it is difficult to say exactly which tax rate is relevant. First of all, not all homeowners itemize their deductions. Secondly, Hendershott and Slemrod [1981] note that the appropriate variable is the average tax savings per dollar due to homeownership. We believe that the marginal rate used here provides a good approximation.

10 Specifically, the decision is based upon the future costs of a unit purchased in the present period. For individuals who finance their entire purchase with a fixed rate mortgage, the borrowing cost component ($r_{mt}$) is constant over the relevant horizon. However, as we noted in Section III.2, in the absence of complete time series data on the aggregate loan to value ratio for new purchases, we set $r_{mt}$ equal to the Aaa corporate bond rate ($r_{ct}$), which does vary over the horizon. The larger the equity proportion of house value, the more accurate will be our approximation. As a practical matter, however, this issue is not of much importance. For historic values of $\gamma_t$ and $r_{mt}$ from 1960-1976, we find that over a typical five-year horizon, the mean difference between the average user cost using our procedure, and one distinguishing between $r_{ct}$ and $r_{mt}$, is only about $48.

A related issue is whether the opportunity cost of funds might be better represented by some other interest rate, such as the return on tax exempt securities. We found that inclusion of the rate on municipal bonds instead
the optimal ARIMA forecasting procedure suggested by Box and Jenkins [1970]. The Box-Jenkins model produces forecasts of a variable based only on past values. Conditional on this information, the forecasts are rational. In principle, one might want to forecast using a completely specified econometric model. This, however, would require forecasting all of the model's exogenous variables into the future. In a similar context, Feldstein and Summers [1978] argue, "There is no reason to expect that the more general procedure that requires estimates of monetary and fiscal policy for many years ahead would yield better forecasts than the simpler Box-Jenkins procedure." (pp. 2-6).

Forecasts made at any given time are based only on information available at that time. (Current year prices are not included in the information set, but all lags are.) Thus, it is necessary to estimate a separate forecasting equation for each year, based upon observations prior to that year. It is not obvious how far into the past the observations for each forecasting equation should go. One possible procedure is to choose some arbitrary length of time (say 10 years) and assume that individuals use data only within that period to make their forecasts. Each year a new observation is added, and simultaneously the observation at the end of the sample is dropped. This method is sometimes called "rolling regression."\footnote{Some justifications for rolling regression are discussed by Friedman [1973]. Feldstein and Summers [1978] use it to generate a time series of expected inflation rates.}

Another possibility is that as more information becomes available over time, individuals employ it, but continue to use the older information as well. Thus, the number of observations grows each year. People believe that the basic economic structure generating the observations remains of little difference. Finally, it might be the case that the relevant opportunity cost might be investment in another house. This possibility is ruled out by our assumption that each individual can own only one dwelling. In a more general model which viewed the housing choice as part of a portfolio decision, this possibility could be analyzed. See Section V below.
the same, but they use new information to update their estimates of the structure's parameters. For practical purposes, a starting point is needed. If World War II is perceived as an important breaking point in economic history, then starting somewhere in the late 1940's is sensible. Essentially, this is no different than the typical practice of using all available post-War data to estimate macroeconomic relationships.

There is not much theoretical basis for choosing between the two assumptions on how information is processed. We tried both and found that the second performed better in the sense of leading to a statistically superior explanation of the tenure choice. The results presented below are based on this method.

After some preliminary analysis of the time series on \( P_t \), we selected an ARIMA (1,1,0) equation to make forecasts in year \( T \):

\[
(P_t - P_{t-1}) = \phi(T)(P_{t-1} - P_{t-2}) + u_t, \quad (t=0, \ldots, T-1)
\] (3.2)

where \( u_t \) is a normally distributed white noise error and \( \phi(T) \) is a parameter to be estimated.\(^{12}\) Again, note that (3.2) is re-estimated each year \( T \) with observations from year 0 to \( T-1 \). Within a given time period \( \phi(T) \) is constant, but as the time period changes, so does \( \phi(T) \). (In practice, year 0 is 1946, and the first \( \phi \) is estimated for 1956.)

Given an estimate of \( \phi(T) \), say \( \hat{\phi}(T) \), equation (3.2) can be solved recursively to generate forecasts of the price of homeownership for as many

\(^{12}\) It is possible to view the ARIMA(1,1,0) model of equation (3.2) as the AR(2) model \( P_t = \phi_1 P_{t-1} + \phi_2 P_{t-2} + u_t \) with the constraint \( \phi_1 + \phi_2 = 1 \). A test on this constraint using observations from 1939 to 1979 indicated that it was consistent with the data—\( F(1,37) = 2.08 \), while the critical level at a 0.05 significance level is 4.08. Note also that with the normality assumption, the distribution of \( P_t \) can be characterized by its mean and variance with no element of approximation.
future years from time \( T \) as desired. This raises the question of the horizon people consider when making their tenure choice decisions. One possibility is that individuals look only to the end of the current year, reasoning that they can always change tenure status after that time. More realistically, substantial transactions costs are involved in moving,\(^{13}\) and one expects that people are concerned about the course of prices at least several years into the future. We assume that people form expectations not only for the current year but four years into the future, and base their tenure choice on the five year average. That is, if we denote \( \bar{F}_{T+5} \) as the simple average of the first five forecasts generated by the \( T \)th version of equation (3.2), then \( \bar{F}_{T+5} \) is entered as the observation for \( F_t \) in equation (2.2). To test the sensitivity of our substantive results to this assumption on horizon length, we also estimated the tenure choice equation assuming that decisions are made on a one-year basis. These results are also reported below.

Figure 3.1 shows the value of \( \hat{\phi}(T) \) for each year. Note that the estimates vary substantially as new information becomes available. Hence, attempts to model expectations formation on the basis of a single ARIMA model estimated for the entire period would likely produce misleading inferences. To the extent there is a trend, the value of \( \hat{\phi}(T) \) tends toward zero. As equation (3.2) indicates, a decrease of \( \hat{\phi}(T) \) in absolute value suggests that relatively more weight is being placed on the most recent observation. This may be due to the increased volatility in \( P_t \) which occurred during the 1970's. This phenomenon, associated mainly with movements in nominal interest rates and capital gains, reduced the value of "old" information. The negative values of \( \hat{\phi}(T) \) suggest that housing expectations are "regressive" in the sense that if prices have gone up in the past, people expect them to return toward some "normal" value.

\(^{13}\) For an estimate of the transactions costs associated with moving, see Venti and Wise [1982].
FIGURE 3.1

ESTIMATES OF PHI OF EQUATIONS (3.2)
Figure 3.2 exhibits for each year the expected price of owner-occupation over a five year period, $\bar{F}_{t+5}$, and compares them to the average of the actual (ex post) prices for the same period. Over the period 1956–1976, the average ex post value of $\bar{F}_{t+5}$ was $562.62; the average ex ante value was $736.14. Due to the nature of the learning process imposed by equation (3.2), individuals react to turning points with a one period lag. Note that in the 1970’s, people often expected the cost of housing to be higher than its ex post value. This may help explain the relatively small change in the homeownership rate during that decade.

It should be noted that our procedure assumes that people form expectations of the real user cost, $F_t$, as a whole. It is also plausible that agents forecast each component of the user cost and then aggregate. The latter procedure, however, is difficult to implement. The investigator must specify and estimate an ARIMA model of each component. Correctly aggregating involves, at a minimum, computing the covariances between separate ARIMA models. The non-linear nature of equation 3.1 complicates matters further. For these reasons, our simpler procedure was adopted for the bulk of the analysis.

Also reported below, however, are estimates based on a model in which real capital gains are the only source of uncertainty. This assumption has been used in earlier studies of tenure choice (Hendershott and Shilling [1982], Rosen and Rosen [1980]) and studies of business investment (Jorgenson [1971]).

3. The forecast error variance of the price of owner occupation ($\sigma^2$).

The same equations used to generate the expected price of owner occupation can be used to produce a series of the forecast error variances. From
FIGURE 3.2

EX POST AND EX ANTE VALUES OF $P_{T+5}$

EX POST (SOLID)
EX ANTE (DASHED)
equations (3.2), at the start of year \( T \), the one year ahead forecast, \( \bar{F}_T \), is

\[
\bar{F}_T = (1+\hat{\phi}(T))P_{T-1} - \hat{\phi}(T)P_{T-2}
\]

(3.3)

The true value one year hence (conditional on 3.2) is

\[
P_T = (1+\hat{\phi}(T))P_{T-1} - \hat{\phi}(T)P_{T-2} + u_T
\]

The error in the one year ahead forecast made at the start of year \( T \) is \((P_T - \bar{F}_T)\), and its variance \( \sigma_T^2 \),

\[
\sigma_T^2 = \hat{o}^2(T) + E \{(\hat{\phi}(T)-\phi(T))(P_{T-1} - P_{T-2}) - E[\hat{\phi}(T)-\phi(T)](P_{T-1} - P_{T-2})\}^2
\]

(3.4)

where \( \hat{o}^2(T) \) is the year \( T \) estimate of the variance of \( u_T \).

Two simplifying assumptions can be made:

(a) The covariance of \( \hat{\phi}(T) \) with the data on which it is estimated \((P_1, \ldots, P_{T-1})\) is zero. To compute it is burdensome, and it is plausible that people ignore this source of error. In this case, equation (3.4) reduces to

\[
\sigma_T^2 = \hat{o}^2(T) + (P_{T-1} - P_{T-2})^2 \hat{o}^2(\hat{\phi}(T))
\]

(3.5)

where \( \hat{o}^2(\hat{\phi}(T)) \) is the estimated variance of \( \hat{\phi}(T) \), computed as usual as

\[
\hat{o}^2(T) = \sum_{i=0}^{T-1} (P_{t-1-i} - P_{t-2-i})^2 / (T-1)
\]

(b) \( \hat{o}^2(\hat{\phi}(T)) = 0 \). This simplifying assumption is made in virtually all ARIMA forecasting. (See Nelson [1973].) Intuitively, it is assumed that there are enough observations so that errors of estimation are of second order importance relative to the inherent uncertainty \( u_T \) in the world. We
then find

$$\sigma_T^2 = \sigma^2(T)$$  \hspace{1cm} (3.6)

Expressions (3.5) and (3.6) give alternative values for the forecast error variance of a one year forecast. Our framework, however, requires computing the variance for a five year average. This leads to two complications:

a. It must be assumed that $\phi(T)$ is known with certainty. Recall that in the case of the one-year forecast, one can choose between assuming that $\phi(T)$ is known with certainty or uncertainty. For the former, equation (3.6) is used; for the latter, equation (3.5) is relevant. Once we forecast further into the future, the computational problem becomes intractable unless we assume that $\sigma^2 \phi(T) = 0$. This is because each forecast error variance contains expectations of third and higher order moments of $\phi(T)$.

In an attempt to gauge the importance of assuming $\phi(T)$ is known with certainty, we estimated two different tenure choice equations with the maintained hypothesis that one-year ahead forecasts were appropriate. In the first $\sigma_T^2$ was estimated using (3.5); in the second, (3.6). The results, which are presented below, indicate that the substantive results are unaffected. Of course, we do not know that this would continue to be the case for the five-year horizon; but the result is suggestive.
b. The variance of the five year average is not simply the average of the five variances. The computation must take into account the covariances between the forecast errors for the various years. Some tedious but straightforward calculations yield the following formula for the five year average forecast error variance,

\[ \sigma^2_{\bar{y}} = \frac{1}{5} \left[ \sum_{i=0}^{4} \alpha_i \right] \]

where

\[ \alpha_0 = \left[ 5 + 4\hat{\phi}(T) + 3\hat{\phi}(T)^2 + 2\hat{\phi}(T)^3 + \hat{\phi}(T)^4 \right] \]

\[ \alpha_1 = \left[ 4 + 3\hat{\phi}(T) + 2\hat{\phi}(T)^2 + \hat{\phi}(T)^3 \right] \]

\[ \alpha_2 = \left[ 3 + 2\hat{\phi}(T) + \hat{\phi}(T)^2 \right] \]

\[ \alpha_3 = \left[ 2 + \hat{\phi}(T) \right] \]

\[ \alpha_4 = 1 \]

Figure 3.3 shows how the five year average forecast error variance changed over time. Its mean value over the sample period was 366,676, and the general tendency has been for it to fall. This is reflective of the pattern of actual prices depicted in Figure 3.2. Although prices in the beginning of the period moved less than those at the end, they did so in a less "predictable" way.

\[ ^{14} \text{Details are provided in an appendix available upon request.} \]

\[ ^{15} \text{A general downward trend interrupted in about 1975 was also found in the forecast error variances generated by the "rolling regression" model.} \]
FIGURE 3.3

FIVE YEAR FORECAST ERROR VARIANCE

[Graph showing the trend of forecast error variance from 1955 to 1979.]
4. The expected value and forecast error variance of the price of rental housing, $\bar{Q}_t$ and $\delta^2_t$.

The same strategy is used to compute $\bar{Q}_t$ and $\delta^2_t$ as was used for $\bar{P}_t$ and $\sigma^2_t$ above. A series of equations of the form

$$(Q_t - Q_{t-1}) = \psi(T)(Q_{t-1} - Q_{t-2}) + \nu_t \quad (t=0, \ldots, T-1)$$

(3.7)

are estimated, and the results are used to generate expected values and variances. Unlike the case of owner occupation, it is not necessary to construct a time series on $Q_t$. Explicit rents are paid to landlords, and data on them are easily available.

Over the period, the ex post real price of renting rises smoothly, as does the forecast value. (The mean ex post value is $1103; the mean ex ante value is $1006.) The forecast error variances of renting are very small compared to that associated with owner-occupation. It seems likely that risks associated with owning are most important to the tenure decision.

5. Other variables

Our theoretical discussion suggested that permanent income should have an effect on housing decisions. Muth [1960, p.30] and others have noted that

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16 Again, this time series process was selected after preliminary investigation.

17 Specifically, $Q_t$ was constructed on the basis of census data on the median rent of renter-occupied units. Further details are provided in a data appendix available upon request. There are, of course, a number of government programs that implicitly and explicitly subsidize rental housing. However, all that matters from our point of view is the market price facing consumers, and this is precisely what the published data are intended to reflect.
current consumption is probably a better "proxy" for permanent income that is current income. We therefore include per capita real consumption, $C_t$, as a righthand side variable.\(^{18}\)

An important issue in the housing literature is the extent to which credit rationing influences housing demand decisions. (See Arceius and Meltzer [1973] and Swan [1973].) A rigorous examination of the impact of credit rationing on the tenure decision would require specification and estimation of a disequilibrium model as suggested by Fair and Jaffee [1972]. A simpler approach is to include among the regressors a measure of the availability of mortgage market funds. For this purpose, we create the variable $CRED_t$, defined as the real growth in deposits at thrift institutions (mutual savings banks and savings and loan associations) between years $t-1$ and $t$. One expects that if credit availability has been a factor in the homeownership decision, then $CRED_t$ will have a positive sign.

IV. Results
A. The Basic Model

In our basic equation, expected prices and their forecast error variances are computed over a 5 year horizon. In terms of equation (2.2), $\bar{F}_t = \bar{F}_{t-5}$, $\sigma_t^2 = \sigma_{t+5}^2$, and $Q_t$ and $\delta_t^2$ are defined analogously. Under these assumptions, and imposing constraints (2.3),\(^{19}\) ordinary least squares estimation of (2.2)

\(^{18}\) The consumption variable includes expenditures on housing. Conceptually, this is appropriate, because the idea is to proxy permanent income, and all components of the consumption stream "belong." Simultaneity is not likely to be an important issue because the dependent variable is a function of the homeownership ratio, not housing expenditures per se. In any case, when consumption net of housing expenditures is used the results are essentially unchanged.

\(^{19}\) Preliminary investigation indicated that the hypothesis that constraint (2.3) is applicable could not be rejected by the data. The F-statistic for the test was 3.28, and the critical value is $F(2, 18) = 3.55$ (5%) or 5.01 (1%). In the unconstrained equation, the coefficients on $C_t$ and $\delta_t^2$ were insignificantly different from zero, a phenomenon likely due to insufficient variability within the sample period. The fact that there were fewer degrees of freedom available in the unconstrained regression may also have contributed to the inability to estimate the coefficients precisely.
\[
\ln \left( \frac{\theta_t}{1-\delta_t} \right) = 0.125 - 4.75 \left[ (\bar{F}_t - \bar{Q}_t) \times 10^{-5} \right] \\
- 6.89 \left[ (\sigma_t^2 - \delta_t^2) \times 10^{-7} \right] + 2.04 \left[ C_t \times 10^{-4} \right]
\]

(4.1)

D.W. = 1.44
\[ R^2 = 0.989 \]

The numbers in parentheses are standard errors. The Durbin-Watson statistic is inconclusive at 5% and does not reject the null hypothesis at a 1% level. In any case, when a first-order correction for autocorrelation is made, the outcome is virtually unchanged.

The coefficient on \((\bar{F}_t - \bar{Q}_t)\) is negative and statistically significant at conventional levels. When the expected excess of the cost of owning over renting increases, the proportion of owner-occupiers decreases. The elasticity of \(\delta_t\) with respect to \((\bar{F}_t - \bar{Q}_t)\) is \(-0.053\). This result is qualitatively consistent with earlier research.

The key new variable introduced in our specification is the difference in the forecast error variances of the costs of owning and renting, \((\sigma_t^2 - \delta_t^2)\). The coefficient on this term is negative and exceeds its standard error by nearly a factor of 4. Greater uncertainty in the price of owning reduces the proportion of homeowners, ceteris paribus. The elasticity of \(\delta_t\) with respect to \((\sigma_t^2 - \delta_t^2)\) is \(-0.188\).

The coefficient of the consumption variable is positive and statistically significant, with an implied elasticity of 0.707. As in previous work using

\[ 20 \text{ All elasticities are evaluated at the average sample values for 1975-79. Because of the substantial volatility in the underlying data, the elasticity calculated for any single year might be misleading.} \]
both cross sectional and time series data, there is a positive relationship
between real per capita permanent income (as proxied by personal consumption
expenditures) and the tendency to choose owner-occupier status.

One potential difficulty with our estimates is that they may be inconsistent
due to simultaneity bias. If increases in the proportion of owner occupiers
drives up the price of owner-occupied housing, then there will be correlation
between \((\bar{P}_t - \bar{Q}_t)\) and the error term \(\epsilon_t\). Recently, Plosser, Schwert and
White [1992] proposed a specification test which can be used to investigate
whether this is a serious problem. Their procedure requires
estimating the model in levels and differenced form. Under the null hypothesis
that \(\epsilon_t\) is i.i.d. and there is no simultaneity bias, the estimates will
be identical. The test statistic, chi-squared distributed with 3 degrees
of freedom, is 1.324, indicating a failure to reject the null hypothesis by
a wide margin.

B. Alternative Specifications

To test the robustness of the basic model several additional specifications
were estimated. In the first, the credit variable (CRED\(_t\)) described earlier
was added to the basic equation. The results are shown in column (2) of
Table I. (Column (1) reproduces the results of the basic equation for convenience.)
The results in column (2) show that the addition of CRED\(_t\) leaves the basic
results essentially unchanged. The CRED\(_t\) term itself is insignificant. At
least in our formulation, the availability of real mortgage credit does not
influence the homeownership decision. As stressed earlier, we do not regard
this as decisive "proof" that rationing is unimportant in the housing market.

The basic model assumed that households used a five-year horizon for
tenure choice decisions. We estimated two alternative equations where a one-
Table I

PARAMETER ESTIMATES*

(Dependent Variable = ln[ \frac{\Theta_t}{1-\Phi_t} ])

<table>
<thead>
<tr>
<th>Variable</th>
<th>(1)^a</th>
<th>(2)^b</th>
<th>(3)^c</th>
<th>(4)^d</th>
<th>(5)^e</th>
<th>(6)^f</th>
</tr>
</thead>
<tbody>
<tr>
<td>(P_t-\bar{P}_t) \times 10^{-5}</td>
<td>-4.75</td>
<td>-4.52</td>
<td>-5.12</td>
<td>-3.71</td>
<td>-5.02</td>
<td>-17.22</td>
</tr>
<tr>
<td></td>
<td>(1.92)</td>
<td>(1.97)</td>
<td>(1.36)</td>
<td>(1.23)</td>
<td>(2.65)</td>
<td>(11.71)</td>
</tr>
<tr>
<td>(\sigma_t^2 - \delta_t^2) \times 10^{-7}</td>
<td>-6.89</td>
<td>-6.87</td>
<td>-7.72</td>
<td>-5.37</td>
<td>6.78</td>
<td>-206.0</td>
</tr>
<tr>
<td></td>
<td>(1.74)</td>
<td>(1.76)</td>
<td>(1.26)</td>
<td>(0.91)</td>
<td>(2.27)</td>
<td>(75.0)</td>
</tr>
<tr>
<td>C_t \times 10^{-4}</td>
<td>2.04</td>
<td>2.10</td>
<td>1.91</td>
<td>2.22</td>
<td>2.36</td>
<td>2.38</td>
</tr>
<tr>
<td></td>
<td>(0.30)</td>
<td>(0.31)</td>
<td>(0.22)</td>
<td>(0.16)</td>
<td>(0.292)</td>
<td>(0.31)</td>
</tr>
<tr>
<td>CRED_t</td>
<td>--</td>
<td>-.0068</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(.0096)</td>
</tr>
<tr>
<td>Constant</td>
<td>0.125</td>
<td>0.114</td>
<td>0.168</td>
<td>0.0084</td>
<td>0.00081</td>
<td>-0.0378</td>
</tr>
<tr>
<td></td>
<td>(0.145)</td>
<td>(0.147)</td>
<td>(0.100)</td>
<td>(0.078)</td>
<td>(0.15)</td>
<td>(0.1482)</td>
</tr>
<tr>
<td>D.W.</td>
<td>1.44</td>
<td>1.41</td>
<td>2.11</td>
<td>2.21</td>
<td>1.29</td>
<td>1.21</td>
</tr>
<tr>
<td>R^2</td>
<td>.989</td>
<td>.989</td>
<td>.993</td>
<td>.993</td>
<td>.987</td>
<td>.956</td>
</tr>
</tbody>
</table>

*Variables are defined in the text. Numbers in parentheses are standard errors. The time period is 1956-1979.

a. Basic equation--5 year horizon.
b. Basic equation with credit variable.
c. 1 year horizon; \phi and \psi known with certainty.
d. 1 year horizon; \phi and \psi uncertain.
e. Basic equation; \bar{V}_t/PL only stochastic component of P_t.
f. Basic equation adjusted for quality differences.
year decision horizon was postulated. The results in column (3) are based on the assumption that the autoregressive parameter in the equation that generates price expectations is known with certainty. Hence, $\hat{P}_t = \hat{P}_t$; $\sigma_t^2 = \sigma_T^2$ of equation (3.6); and $\delta_t$ and $\delta_T^2$ are computed analogously. Column 4 is based on the assumption that the autoregressive parameter is uncertain---$\sigma_t^2 = \sigma_T^2$ of equation (3.5), and $\delta_t^2$ is computed analogously.

Taken together, the results of columns (3) and (4) show that: (i) the basic estimates of column (1) are not very sensitive to reasonable changes in the time horizon, and (ii) neither are they sensitive to the assumption that the autoregressive parameter in the price expectations equations is known with certainty.\footnote{We also estimated an equation in which the planning horizon for owning was five years, and the horizon for renting was one year. These results, which are available upon request, are essentially identical to those reported in column (1).}

Column (5) shows the results when the user cost of home owning is computed under the "traditional" assumption that the only unknown component is the expected real capital gain. Specifically, we estimated a series of ARIMA models for real capital gains, and used them to compute the expected value and forecast error variance over 5-year horizons, just as was done for the entire user cost in Section III. The other components of $P_t$ were assumed known with certainty. As the results indicate, not much changes. This is not too surprising, since much of the variability in the $P_t$ series is associated with changes in house value.

Finally, we estimated a version of the model trying to take into account changes in the qualities of owner-occupied and rental housing over the period. The only dimension of housing quality for which time series data are available is the average size of rental and owner-occupied units. Column (6) reports results when the user costs were scaled by average number of rooms for owner-occupied and rental housing. (A five-year horizon is again assumed.) The qualitative results are similar to those previously obtained, although the
coefficients differ as a result of the scaling. Of course, this is a crude adjustment for quality change, but it is the only one available over the time period.

C. Some Implications

To get a better feel for the quantitative significance of our results, it is useful to employ them as the basis for a number of simulations. Typically, simulations of the impact of changes in the housing environment focus exclusively on the effects upon the user cost of housing. However, any exogenous force which changes mean expected prices will also affect the forecast error variances. To accommodate this problem, the following simulation procedure was adopted:

1) A counterfactual was posed. For instance, "What would have been the effect upon the homeownership ratio if the growth rate of real house values had been constant over the sample period?" (discussed below)

2) An artificial ex post user cost series was calculated by evaluating equation (3.1) under the counterfactual hypothesis.

3) Equations (3.2) were re-estimated on the artificial data series, resulting in new estimates of \( \delta(T) \). These were used to calculate expected prices of home ownership and forecast error variances under the counterfactual.

4) The counterfactual series of price differentials and forecast error variance differentials were substituted into the estimated behavioral equation (equation (4.1)) to predict the homeownership ratio which would have obtained under the counterfactual. To avoid peculiarities associated with any particular year, comparisons of actual and simulated homeownership ratios are presented on the basis of 5-year averages over 1975-1979.

The first proposition considered was the effect of a constant growth rate in real house values \((V_t / P_t)\) in equation (3.1)). To investigate this, we created an artificial series whose endpoints matched the historical record,
but which grew smoothly at the rate of 3.1% yearly, and then followed the
simulation procedure outlined above.

The results indicate that a steady real growth rate in housing prices
would have increased the proportion of owner-occupiers in the late 1970's
by +0.0334. It is useful to "decompose" this figure into the parts due to
the change in the expected price difference, and the part due to the change in the
difference in the forecast error variances. If $\sigma_t^2$ is held at its actual
value and the artificial value of $\bar{p}_t$ is substituted into equation (4.1), we
find that the proportion of owner-occupiers falls by 0.0072. Under the
simulation, capital gains in the latter part of the period are smaller than
historical values, so on the basis of expected price alone, owner-occupation
is less desirable than it was in reality. On the other hand, if $\bar{p}_t$ is
held at its actual value, and the artificial value of the forecast error
variance is used, then the proportion of owner-occupiers increases by 0.0406.
Clearly, the encouraging effect of less uncertainty dominates the outcome.
For reference, these results are recorded in column (1) of Table II.

We next gauged the impact of several proposed changes
in the tax treatment of housing. Suppose that during our sample period the
deduction of mortgage interest and property taxes had been disallowed, but
everything else had been the same. The results are recorded in column (2)
of Table II. Elimination of these deductions would have decreased the
proportion of owner-occupiers by 0.0040. Most of the effect (.0036) comes
via changes in the expected price; elimination of the deductions does not
have much impact upon the forecast error variance. 22

---

22 The magnitude of this effect is somewhat smaller than that found in
earlier studies such as Hendershott and Shilling [1982] and Rosen and Rosen
[1980]. This is due in part to the fact that the marginal income tax rates
used in those studies exceed those computed by Joines [1981], which are the
ones used here. Hence, removal of any given tax deduction has a smaller
dollar effect on the user cost of housing in this paper than in its predecessors.
TABLE II
CHANGE IN AVERAGE PROPORTION OF HOMEOWNERSHIP
DURING 1975 - 1979*

<table>
<thead>
<tr>
<th></th>
<th>Constant Growth Rate of Housing Prices (1)</th>
<th>No Deductions (2)</th>
<th>Capital Gains Taxed (3)</th>
<th>No Deductions, Capital Gains Taxed (4)</th>
<th>Flat Tax, Current Treatment of Housing (5)</th>
<th>Flat Tax, No Deductions (6)</th>
<th>Flat Tax, No Deductions, Capital Gains Taxed (7)</th>
<th>Indexing (8)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Effect</td>
<td>+.0334</td>
<td>-.0040</td>
<td>+.0105</td>
<td>+.0063</td>
<td>+.0031</td>
<td>-.0040</td>
<td>+.0079</td>
<td>-.0002</td>
</tr>
<tr>
<td>Expected Price Effect</td>
<td>-.0072</td>
<td>-.0036</td>
<td>-.0051</td>
<td>-.0089</td>
<td>+.0025</td>
<td>-.0036</td>
<td>-.0100</td>
<td>-.0002</td>
</tr>
<tr>
<td>Forecast Error Variance</td>
<td>+.0406</td>
<td>-.0004</td>
<td>+.0156</td>
<td>+.0152</td>
<td>+.0006</td>
<td>-.0004</td>
<td>+.0179</td>
<td>+.0000</td>
</tr>
</tbody>
</table>

*All comparisons are relative to the average fitted value of the basic equation for 1975-1979, 0.6833. The proportions are adjusted for changes in household composition as described above.
Another tax reform possibility is to tax housing capital gains at ordinary rates, but leave the other deductions in place. As shown in column (3) of Table II, this change would have increased the proportion of owner-occupiers by 0.0105. This is a surprising result, but the other figures in column (3) reveal its source. On the basis of expected price alone, we would have predicted a decrease of -.0051. However, the encouraging effect of the tax-induced reduction in the forecast error variance dominates the outcome. This kind of result is familiar from the literature on taxation and uncertainty (Tobin [1958]). As far as we know, its relevance to the issue of housing tenure choice has not been established before.

Column (4) records the results when the interest and property tax deductions are removed and housing capital gains are taxed. On balance, there is a very small increase in the proportion of owner-occupiers. The variance effect so strongly present in column (3) is mitigated to some extent by the expected price effect of column(2).

Recently, a number of suggestions have been made to change the current income tax into a "flat tax." Generically, a flat tax is an income tax with a constant marginal tax rate and a broadly defined tax base. However, a number of quite different proposals have been labeled as flat taxes. In particular, the various proposals have differed substantially with respect to the tax treatment of housing—some would leave it virtually unchanged, while others would eliminate most of the current benefits. Clearly, one cannot simulate the impact of "the" flat tax. We therefore simulate the effects on tenure choice of three different proposals:
a) Institute a tax with constant marginal rate of 24%, but which allows the current deduction of interest payments and property taxes, and which continues the current exemption of housing capital gains. The figure of 24% is based on Slemrod and Yitzhaki's [1983] estimate that it would best balance efficiency costs with minimal changes in the distribution of tax burdens. According to the results in column (5), there would not be much of an impact on the homeownership ratio. This is due mostly to the fact that the flat rate of 24% does not differ all that much from the average of historical marginal tax rates. More disaggregated data would allow us to examine how differential changes in tax rates within the population might affect the tenure choice. Unfortunately, such data are not available.

b) Stipulate a constant marginal tax rate of 0.24, but eliminate the mortgage interest and property tax deductions. However, capital gains are still untaxed. As indicated in column (6), this regime induces a slight decrease in the incidence of homeownership, most of which is due to the increase in the user cost associated with the loss of the deductions.

c) Institute a constant marginal tax rate of 0.24, eliminate interest and property tax deduction, and tax housing capital gains. The results, in column (7), show a slight stimulative effect on the homeownership ratio. This result is not unexpected given our previous discussion of column (3)--taxing capital gains makes housing more expensive, but this effect is overwhelmed by the reduction in risk associated with the tax's introduction.

Finally, we consider briefly the effects that indexing the income tax might have upon the homeownership ratio. From the point of view of constructing a counterfactual time series on the user cost of owning, indexing presents
enormous difficulties. What real rates would have been built into the tax system? Would deductions and exemptions have been computed in real terms as well? If so, what price index would have been used?

In the absence of any obvious way to answer these questions, a reasonable procedure is to assume that the ultimate effect of indexing would simply be somewhat lower marginal tax rates. In column (8), we assume that under indexing, marginal tax rates would have been 10 per cent smaller in each year than the historical values. The results suggest a very small depressing effect on the homeownership ratio--when the marginal tax rates decreases, the tax advantages associated with homeownership are worth less, so the user cost increases.

V. Conclusions

In this paper, we investigate the effects of price uncertainty on the tenure choice decision. Estimates on data from 1956-1979 indicate that uncertainty over the course of relative prices has significantly depressed the aggregate proportion of homeowners.

Proposals to modify the tax treatment of housing affect both the expected price differential between renting and owning and the difference in the forecast error variances. Previous analyses of policy changes may be misleading because the two effects can work in opposite directions. For example, our results suggest that taxation of capital gains at marginal personal income tax rates would have increased the homeownership rate, despite the increased
expected cost of owning. The reduction in price variance is sufficiently attractive to dominate the outcome. Other results indicate that eliminating property tax and interest payment deductions would have reduced the homeownership rate, but that the combination of no deductions with capital gains taxation would have resulted in a slightly higher proportion of homeowners.

These results provide some explanation for the puzzling behavior of the homeownership ratio in the late 1970's. In that period *ex post* costs of homeownership fell greatly relative to renting. Despite this, the aggregate proportion of homeowners changed little. Our evidence suggests that this was largely due to the erratic nature of housing costs which made ownership commitments unattractive.

The chief limitation of this analysis is its omission of the relationship between housing and other financial decisions. From a theoretical point of view, one expects that the housing decision will be part of a broader portfolio allocation problem. As an empirical issue, the relevance of this consideration is not clear -- in 1972, 80% of those under the age of 55 held less than $5000 in financial assets. (Feldstein and Feenberg [1983]). Nevertheless, this is a topic worthy of further investigation.
Appendix A

This appendix details the calculations of the forecast error variance of a projection based on the simple average of the first five future observations.

We can write the process for generating prices as:

$$P_{t+j} - P_{t+j-1} = \phi(P_{t+j-1} - P_{t+j-2}) + u_{t+j}, \quad (A.1)$$

where $\phi$ is assumed to be known.

Using the lag operator, (A.1) implies:

$$(1-L)P_{t+j} = \phi(L)(1-L)P_{t+j} + u_{t+j}$$

or

$$(1-\phi L)(1-L)P_{t+j} = u_{t+j}$$

Finally,

$$P_{t+j} = \frac{1}{(1-L)(1-\phi L)} u_{t+j} \quad (A.2)$$

Expanding (A.2) yields an expression for $P_{t+j}$ as a weighted average of past shocks:

$$P_{t+j} = \sum_{k=0}^{\infty} c(k) u_{t+j-k} \quad (A.3)$$

where

$$c(k) = 1 + \phi + \phi^2 + \ldots + \phi^k \quad (A.4)$$

$$c(0) = 1$$

The expected price, $\hat{P}_{t+j}$, at time $t$ is calculated by taking the
expectation of (A.4) conditional on information known at time \( t \) (which excludes \( u_t \)):

\[
\hat{p}_{t+j} = \sum_{k=j+1}^{\infty} c(k) u_{t+j-k}
\]

Thus, the forecast error is:

\[
e_{t+j} = p_{t+j} - \hat{p}_{t+j} = \sum_{k=0}^{j} c(k) u_{t+j-k}
\]

Let \( w_0, w_1, w_2, w_3, w_4 \) be weights. Then the 5 period weighted average forecast error is:

\[
\overline{e} = \sum_{i=0}^{4} w_i e_{t+i}
\]

\[
= w_0 \sum_{k=0}^{0} c(k) u_{t-k} + w_1 \sum_{k=0}^{1} c(k) u_{t+1-k} + w_2 \sum_{k=0}^{2} c(k) u_{t+2-k} + w_3 \sum_{k=0}^{3} c(k) u_{t+3-k} + w_4 \sum_{k=0}^{4} c(k) u_{t+4-k}
\]

Collecting coefficients on the shocks:

\[
\overline{e} = u_t \sum_{i=0}^{4} w_i c(i) + u_{t+1} \sum_{i=0}^{3} w_{i+1} c(i) + u_{t+2} \sum_{i=0}^{2} w_{i+2} c(i) + u_{t+3} \sum_{i=0}^{1} w_{i+3} c(i) + u_{t+4} w_4 c(0)
\]

Clearly \( E(\overline{e}) = 0 \). Thus \( \text{Var}(\overline{e}) = E(\overline{e}^2) \). Since the \( u \)'s are independently, identically, distributed, all covariance terms disappear and the result is:

\[
\text{Var}(\overline{e}) = \sigma_u^2 \sum_{i=0}^{4} w_i^2 a_i^2
\]

where:
\[ a_0 = \sum_{i=0}^{4} w_i c(i) \]
\[ a_1 = \sum_{i=0}^{3} w_{i+1} c(i) \]

In the case referred to in the text, \( w_i = 1/5 \) for all \( i \).
Appendix B

This appendix describes the methodology used to construct our data series. The source of our raw data is also documented. HS refers to Historical Statistics of the United States: Colonial Times to 1970 (1975) and SA refers to various editions of Statistical Abstract of the United States.

1. Proportion of owner-occupied dwellings, adjusted for demographic composition: $\theta_t$.

   Three different sources of data were utilized. From 1945 to 1959 the iterative perpetual inventory method described in Rosen and Rosen (1980) was used. From 1960 to 1973 data on the proportion of owner-occupied housing starts were taken from Housing Vacancy Survey, U.S. Bureau of the Census, Publication #111. From 1974 to 1980, the owner-occupancy data were taken from the Annual Housing Survey, U.S. Bureau of the Census and Department of Housing and Urban Development. The owner-occupied rates were then adjusted for changing demographic composition of the population following Jaffee and Rosen (1979).


   As noted in the text, the price of owner-occupied housing is:
   \[ P_t = \frac{(1-\tau_t)[Y_t] V_t + (1-Y_t) T_t V_t + T_t + D_t + M_t - V_t}{PL_t} \]
   where $\tau_t$ is the AAA bond rate, $V_t$ is the market value of a house, $T_t$ is the property tax per single family housing unit, $D_t$ is depreciation, $M_t$ is maintenance, $V_t$ is the expected capital gain, $\tau_t$ is the marginal tax rate for the household with average taxable income, $Y_t$ is the share of owner's equity in the house, and $PL_t$ is the implicit price deflator for total consumption expenditures.

   "It was assumed $\tau_{ct} = \tau_{mt} = \tau_t$.\]
The AAA bond rate series was taken from HS and SA. Following actual real estate practice, depreciation and maintenance were each set at 1 percent of the house's value. The tax rate \( t \) was taken from Joines, [1981, p. 210]. After 1975, the 1975 tax rate of .1479 was used.

The market value of owner-occupied housing, \( V_t \), was derived by first splicing two housing price series and using the results to compute annual rates of change of house prices, \( g_t \). The values of \( g_t \) were then applied to census-year numbers on the median value of owner-occupied units in order to derive an annual series comparable with the census-year numbers. In an iterative process, the values of \( g_t \) were changed proportionately until the values of the constructed price series for census years exactly matched those of the census. Median values of owner-occupied units in census years were found in HS. For 1944 to 1966, \( g_t \) was computed using FHA sales price data as reported in various editions of the FHA Yearbook. For 1967 to 1980, \( g_t \) was calculated from various editions of Existing Home Sales, a publication of the National Association of Realtors [1980].

The property tax per owner-occupied unit was calculated by dividing the residential portion of all federal, state, and local property taxes by the number of owner-occupied units:

\[
T = \frac{[KR_t/(KN_t + KR_t)] \times PTT_t}{OS_t}
\]

where \( PTT_t \) is total property tax revenue, \( KR_t \) is net private residential capital stock at current cost, \( KN_t \) is net private nonresidential capital stock at current cost, and \( OS_t \) is the number of owner-occupied units. For years prior to 1971, these series were taken from HS; for 1971 to 1980, they were from SA.
3. Price of rental housing: $Q_t$.

An annual rate of change of the rental price of housing was computed using the rental component of the CPI (HS prior to 1971, SA for 1971 to 1980). This rate of change series was then applied to census-year numbers on the median rent of renter-occupied units in order to derive an annual series comparable with the census-year numbers. In an iterative process, the annual changes in rental prices were adjusted until they exactly replicated the census-year numbers.

4. Real consumption: $C_t$.

Real per capital consumption expenditures were taken from the *Economic Report of the President* (1982).

5. Real growth in deposits at savings and loan associations and mutual savings banks: $\text{CRED}_t$.

The savings data were from HS for the 1949 to 1970 and from SA for 1971 to 1980.

6. Quality adjustment for rental and owner-occupied housing.

An annual time series on the number of rooms in renter and owner-occupied housing was developed from census data prior to 1973, and from the *Annual Housing Survey* since 1973. The price variables for renter and owner-occupied housing were then recalculated on a per room basis.
References


Hendershott, Patric H. and Joel Slemrod, "Taxes and the User Cost of Capital for Owner-Occupied Housing," mimeo, The Ohio State University, 1981.


